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Implementing Environmental Education in the *Curricular Strategy* at Tertiary Education from the Perspective of Mathematics

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Abstract: Environmental education has become a vital topic in schools nowadays. From primary school to tertiary education, environmental education can be taught in every subject because of its versatility. However, in some universities this is not the case and some students ignore how their majors can contribute to raise their environmental education. The present work is directed to improve the performance of the students at the university level by designing and implementing teaching tasks to raise both their culture and their environmental education from the perspective of Mathematics. Some methods were applied in order to analyze the current design of the *Curricular Strategy* used in tertiary education and to determine how environmental education should be implemented in the *Curricular Strategy* without affecting its structure. Problems of differential equations from the discipline of Higher Mathematics were used as an alternative supported by a psycho-pedagogical approach. The exercises achieved the purpose stablished by the authors. Teaching tasks based on everyday life allow the students to understand the essence and get a more global vision of the object of study. This research was considered as a valid contribution for the confection of new curricular strategies and it also served as a basis to inspire some other works from the perspective of Physics and Biology.

Keywords: Environmental Education, *Curricular Strategy*, Higher Mathematics, Teaching Tasks

1. Introduction

Education is a social phenomenon, the result of the historical development achieved, exerting a decisive influence on the formation of man, preparing him for the enjoyment and fullness of everything that derives from it, according to the society in which he lives. It has to achieve the difficult balance of offering an educational and comprehensive response, providing a common culture to all individuals in society. That is why it is necessary to provide each one with what he or she needs in order to maximize his or her possibilities and identity. Then, it is necessary to prepare students in the scientific conception of the world, that is to say, to develop in all human fullness the intellectual, physical and spiritual capacities.

In this sense Mathematics plays an important role, its influence is felt not only in the sciences but also in teaching, which has among its central objectives to offer every citizen a solid mathematical preparation [1].

In the context of Mathematics teaching, one of the aspects that attracts most attention is problem solving as a teaching task, which is one of the guiding objectives in all teaching. Working with such tasks should not only consolidate mathematical knowledge, but also develop the student's personality, i.e. the development of his or her scientific conception of the world and of an active and critical position with respect to phenomena, natural and social facts. For this purpose, it is necessary to properly select the exercises through which it is possible to act on a certain sphere of the pupil's personality and to exploit its content to the maximum from the development of the educational function. One of these attentions is Environmental Education from Higher

Mathematics at the University level.

The coherent work of the school with the other educational forces is an indispensable condition for the development of environmental education as an alternative solution to environmental problems, in order to instruct and educate students towards the rational use of natural and socio-economic resources of the environment. Universities are not exempt from this task; on the contrary, they must plan actions to offer treatment to environmental problems from the initial training of future professionals. In *Cuba's National Strategy for Environmental Education*, it is stated that it is necessary to: "Introduce the environmental dimension with an interdisciplinary character, in the models of the professional, study plans in their academic, work and research components, as well as in the science and technical plans of the National System of Higher Education [2]."

As can be seen, environmental education is a necessity in order to face current environmental problems and to achieve this it is necessary to have teachers prepared to deal with them. For this reason, in the initial training of university professionals, attention should be paid to their preparation for environmental education and not wait until they graduate to carry out actions in this direction. The current policy in the formation of professionals is aimed at responding to the transformations that have been taking place in Higher Education, focused on -developing the quality of education in both the formative and instructional aspects and contributing to a comprehensive general culture. In this order, the universities must carry out educational and research actions to improve environmental management and to form an adequate professional, which requires that he/she demonstrates by example and action the system of knowledge, scientific methods and values, which allows him/her to manifest an environmental culture for sustainable socio-economic development.

For this they must be governed, first of all, by *the 2030 Agenda and the Sustainable Development Goals*, in which those related to this work are selected: "15.5 Take urgent and significant action to reduce the degradation of natural habitats, halt biodiversity loss and, by 2020, protect threatened species and prevent their extinction; 15.7 Take urgent action to end poaching and trafficking of protected species of flora and fauna and address both the demand and supply of illegal wildlife products; 15.c Increase global support for combating poaching and trafficking of protected species, including by increasing the capacity of local communities to pursue sustainable livelihood opportunities [3]".

All of the above must be considered when elaborating the teaching tasks to be proposed to students in the development of teaching, in correspondence with the content of each objective and its purpose to be achieved.

Another element to consider in order to achieve the link between Higher Mathematics and Environmental Education is related to interdisciplinarity. To this end, we assumed the definition of interdisciplinarity expressed by [4] in which it is approached as a process, a way of thinking and proceeding, which allows man to know the objective reality, to be able to solve any of the complex problems it poses.

Another of the aspects dealt with revolves around the conditions to make interdisciplinarity possible, also assumed by the same author, who states that these conditions are based on the competence that each teacher must possess and the interest and motivation to carry out interdisciplinarity, the efficient methodological work that the institution must develop, where the management and technical bodies play a predominant role, which are responsible for directing the methodological work under the influence of community factors, together with the methodological preparation that the universities must provide both in undergraduate and postgraduate studies with the aim of developing the interdisciplinary approach in teachers as a work philosophy.

About the advantages offered by the development of interdisciplinarity, which are also assumed by the researchers, the importance of this process was analyzed since it eliminates the borders between disciplines, makes possible the increase of motivation in students, as well as an interrelation between the concepts that are developed, and the definitions with which the student operates, allowing, also the systematization of skills and educating a logical, reflective and integrative thinking at the same time that awakens the interest of teachers and propitiates among them better working relationships. However, after all that has been stated above the authors found that few investigations have been conducted regarding the incorporation of environmental education into the *Curricular Strategy*. Therefore, this paper proposes a set of teaching tasks related to environmental concerns which can be solved from the perspective of Mathematics applying an interdisciplinary approach.

1.1. Introducing Environmental Education into the Curricular Strategy for Achieving Sustainable Development

Humanity has to work sustainably to solve the various problems of a global nature, on the solution of which the existence of the human species depends; and thus the need for Environmental Education which is understood as a continuous and permanent process, which constitutes a dimension of the integral education of all citizens, oriented to the acquisition of knowledge, development of habits, abilities, capacities and attitudes and in the formation of values, which harmonize the relations between human beings and between them and the rest of society and nature, to propitiate the orientation of the economic, social and cultural processes towards sustainable development [5].

The concept of curriculum is assumed as the process of selection, organization and distribution of the knowledge considered valid to achieve the learning goals. Attention should be paid to the curricular plan which is essential for any learning process since it starts from the real needs of the context allowing a strong connection between reality and education [6]. *Curricular Strategies* are designed to be the leitmotiv of the teaching and learning process [7]. "*The Curricular Strategies* ensure the achievement of objectives that, due to their scope, go beyond the possibilities of a discipline, and therefore, must be assumed by all or part of them. They are specified in each of the years of the course, as part of its objectives [4]".

Environmental education can be taught by selecting local environmental situation to contextualize curriculum making content more meaningful and encouraging students to change their reality. Also, by addressing environmental education from different educational approaches. This evidence the possibility for teachers to develop environmental education from their diverse profiles and experience. Environmental education is not an issue to only be addressed by science teachers or the didactic of science, but all teachers and all educational research fields. Another example could be the integration of knowledge for reading/transforming reality. Its purpose is to understand the context from the multiplicity of disciplines and interdisciplinary perspectives. Reading and approaching environmental issues requires more complex perspectives, such as interdisciplinarity, which provide greater comprehension and intervention capacity than isolated disciplines [8].

The Environmental Education Strategy proposes the following objectives:

a) strengthen the incorporation of environmental education for sustainable development (EEfSD), integrating health, energy and savings education in the initial training process of professionals,
b) educate professionals in training to reduce the impact of disasters on the environment,
c) strengthen the EEfSD, attending to the different components of the formation process, in a coherent, systematic and integrated manner, incorporating the university's student organizations,
d) to promote the participation of the mass media in the development of environmental education for professionals in training,
e) to guide, advise and monitor student scientific activity in relation to environmental education,
f) systematically assess the levels of participation of trainees in the EEfSD, based on defined indicators.

Within its Theoretical Guidelines are the Basic Concepts, which are mentioned below: environment, sustainable development, environmental problem, environmental education, environmental education for sustainable development (EEfSD), environmental professional performance and initial environmental training of professionals.

In the body of the *National Environmental Education Strategy (*2010-2015) the following are detailed as Prioritized Themes: climate change, hazard, vulnerability and risk, sustainable use of water resources, conservation and sustainable use of biological diversity, sustainable land management, combating environmental pollution, safe management of chemicals and hazardous waste, sustainable consumption and production, coastal zone management, environmental law and citizen participation, protection of natural and cultural heritage. Within the Methodological Guidelines we can name the Pedagogical Requirements:

a) The unity of the natural and social environment.
b) The unity of the affective, the cognitive and the attitudinal approach.
c) The development and transformation of attitudes.
d) Unity between the local, national, regional and global approach.
e) Improvement of the quality of life.
f) The unity between environment and development.

The Contents take into account the following guiding ideas: a) human beings, as an integral part of the environment, develop in constant interrelation with nature, b) the improvement of the quality of life of human beings, constitutes the fundamental ways that ensure the survival of present and future generations, c) human beings ensure the maintenance of the conditions of the planet with a responsible attitude towards the environment.

In the objectives for each year of the Professional Model, it is stated in common for the first year of university courses: to describe the relationship of man as a biopsychosocial being, in contemporary environmental problems at different scales, so that professors can stimulate ways of thinking, feeling and acting responsibly with the environment in their students. That is why university teachers must take into account this purpose for the elaboration, design and implementation of teaching tasks aimed at this purpose.

1.2. Implementing Teaching Tasks in the Environmental Education Curricular Strategy

The ways to achieve Environmental Education from Mathematics also require the design and application of teaching tasks as a complement and base on which the linking of knowledge must be sought, and this constitutes a challenge and at the same time one of the main directions of methodological work in the subjects. "The main challenge of our educational systems of tomorrow is the tension between, on the one hand, a generalist type of teaching, based essentially on the development of knowledge and skills, whose goal is above all to provide the student with knowledge, which may be powerful in the long term, but not very operational, and on the other hand, a specific type of teaching, more functional, based more on the development of skills that lead the student to reinvest knowledge in meaningful situations [9]."

In this sense, one of the teaching activities of great contribution is the teaching task in function of the formation of the professional, accentuated at the present time by the need to form in the students' essential qualities of cognitive independence and creativity together with the necessary preparation for the solution of problems of their profession.

Several authors have admitted that the teaching task is the cell of the teaching-learning process [10]. For these authors, the teaching task begins to develop with the formulation of the problem and is considered as the cell, the genetic nucleus of independent work, so that great attention must be paid to its formulation. With the task oriented by the teacher, each student has to reflect his or her needs, motivations and interests, which makes it possible to demonstrate assimilated knowledge, developed skills and values in formation. The teaching-learning process becomes more individualized and personified.

Therefore, it can be said that the purpose of the teaching task is to develop skills, conceived as a dynamic and integrated process of knowledge, skills, attitudes and values, which can be induced and developed during the teaching process, and their degree of appropriation should be evaluated.

"The teaching task is the task that includes the contents of the different disciplines and once they are dialectically assimilated in their cognitive structure, it makes it possible for the student to apply them in their practical activity [11]."

The following are requirements for the design and application of the teaching task for training purposes:

a) include contents from several subjects or disciplines,
b) to promote the formation of skills, abilities,
c) the contents must be linked to professional problems,
d) the student must perceive its usefulness (why it will be done, for what purpose, what it contributes),
e) it must contain the precise guidelines to solve them and time of development,
f) should have variants to broaden individual and team participation of students. For this, it is relevant the planning, organization, direction and control of the work, with a good orientation and structured in stages and phases of development of the different tasks to execute in the solution of the problem, to reach the objectives and to form the abilities.

The authors of this work define the following actions: 1) The first thing to be achieved is the environmental diagnosis and of the needs for the environmental professional performance of students, as a starting point; 2) The understanding, on the part of teachers and managers of each faculty, in particular, of the need to incorporate the EEfSD to the process of teaching and learning mathematics at the university; 3) The preparation of the teachers and the directors of each faculty for the incorporation of the EEfSD to this process, from the methodological work system, in each level; and 4) The motivation of the students in formation and of the leaders of the student organizations to incorporate the EEfSD to their daily work.

For designing the teaching tasks it should be considered that: motivation can be increased if learning situations are derived from the observation of different aspects or themes that can be illustrated through the projection of videos, or other resources that become attractive media for students; the supporters of this point of view believe that the environmental knowledge of the different phenomena presented would awaken the interest and the change of attitudes of students for the mathematical contents object of teaching.

In order to design teaching tasks with the purpose of using Environmental Education in the teaching of Mathematics, it is necessary to take into account not only the content, but also the psycho-pedagogical conception on which the teaching-learning process is based. In the particular case of the Cuban university, this process is based on the developmental approach.

2. Materials and Methods

In the course of this research a qualitative methodology was applied where the following strategies were applied:

1) Document analysis: This was used to determine the methodological basis of the research and how each document contributed to it.
2) Participant observation: It allowed the identification of existing problems.
3) Interviews: It exposed the lack of environmental culture that students have because it is not reflected in the *Curricular Strategy*.

3. Results and Discussion

According to what has been analyzed in this work, the authors considered it necessary to highlight a group of aspects that must be taken into account in the design of the tasks. These aspects are:

1) The general majors' objectives, the specific year objectives, and the objectives of each of the disciplines.
2) The invariants of each of the disciplines including the *Environmental Education Strategy.*
3) The systemic approach to tasks as a particular case of teaching tasks.
4) The sequentiality of the tasks. This aspect refers to the order in which the tasks will be presented. The degree of complexity and difficulty of the tasks should be taken into account.
5) The number or volume of tasks to be targeted considering the output of the *Environmental Education Strategy.*
6) The moment in which they will be oriented. In this aspect, the logic of the development of the contents must be considered, so it is necessary to determine the contents assimilated by the student until the moment in which they are going to be oriented, so that they can make use of them in their solution.
7) Indicators to be taken into account for the evaluation of the tasks.

3.1. Didactic Suggestions for Working with the Teaching Tasks

1) There are many teaching aids available to a teacher today in Cuban universities, from the traditional blackboard, books, newspapers, photos, diagrams, graphs, charts, tables, drawings, paintings, models, maps, television, radio, recordings, etc. All of them are opportune and necessary as long as the teacher uses them correctly, in a timely manner and achieves meaningful and developmental learning in his or her students.
2) None of the aspects in the previous section can be ignored when designing the teaching tasks to be set for

the students. It is important that in each of the tasks the teachers take into account the objectives of the 2030 Strategy, the objectives of the *National Environmental Education Strategy* document and that of each problem to be worked on, as well as highlighting the educational aspects of what corresponds to the environmental part, both on the part of students and teachers, and exploiting the potential of the task in question to the maximum.

3) To encourage the students to express their open ideas about the subject being addressed in the teaching task without being interrupted by any classmate, always rectifying their technical and scientific vocabulary, and taking into account the deficiencies detected in the fine diagnosis to contribute in this way to the eradication of their possible errors and their overcoming.

3.2. Memento on Differential Equations of Higher Mathematics for Solving Teaching Tasks

Many population growth problems can be modeled by applying mathematical tools. Population growth problems lead to equations that are known as the Lotka-Volterra or Predator-Predator-Prey system equations, other problems can be of the recursive type. These equations have the form:

$$\frac{dR}{dt} = kR - aRW$$

$$\frac{dW}{dt} = -rW + bRW$$

k, r, a, b are positive constants representing the interactions between predators and prey and t is time. The term aRW determines the rate at which prey decreases and bRW is the rate at which predators increase. Let's call R for the amount of prey in an area and W, as the amount of predators in that area [12].

Referring to the solution of recursive equations it is necessary to rescue a series of aspects that were studied in the work of differential equations and those are:

1) General solutions of a differential equation.

2) Work on differential equations with initial values.

A recurrent equation has the form $c_n a_n + c_{n-1} a_{n-1} + \ldots + c_{n-k} a_{n-k} = f(n)$; $k \in Z^+$ and each $c_i \neq 0$ (constant) $n \geq k$. The relation is defined as homogeneous if f(n)=0 otherwise it is defined as inhomogeneous; it is defined linear if there are no squared terms or no products of a_j. When a recurrence is solved, three cases can be presented depending on the solutions obtained; they may not be repeated, they may be repeated or they may be complex results; something that will not be considered as a special case is the change of variable that is used to solve recurrent equations a little more complex taking it to a simple form, however, it will be addressed to have an idea of how to use it. The authors propose the reader to review the method of solving non-homogeneous recurrent relations in the books *Discrete and Combinatorial Mathematics and Applied Introduction* by Ralph Grimaldi, 2004. The methods for solving recursive relations are equivalent to the methods used to solve differential equations, most of the methods that will be discussed here are derived from working with indeterminate coefficients in a differential equation. The solutions by either a recursive or differential equation will coincide if the problem is well modeled.

Case 1: $a_n = 2a_{n-1} + 3a_{n-2}$ with $a_0=1$, $a_1=2$ [13]

The general solution is obtained by finding its characteristic polynomial P(x) and its roots r 1and r 2. To get to the characteristic polynomial we can make the substitution $a_n = cr^n$, that would lead us to:

$$cr^n - 2cr^{n-1} - cr^{n-2} = 0 \ /{:}cr^{n-2}$$

$$r^2 - 2r - 3 = 0$$

$$P(x) = r^2 - 2r - 3 = (r-3)(r+1);\ r_1 = 3,\ r_2 = -1$$

$$a_n = c_1 3^n + c_2(-1)^n$$

At this point it should be noted that the above equation is a general solution so that for different values of c_1 and c_2 infinite solutions are obtained. To find the specific solutions it is carried out:

For n=0 and $a_0=1$, we obtain: $c_1 3^0 + c_2(-1)^0 = 1$ or $c_1 + c_2 = 1$

For n=1 and $a_1=2$, we get: $c_1 3^1 + c_2(-1)^1 = 2$ or $3c_1 - c_2 = 2$

After solving the system we obtain the values of:

$c_1 = 3/4$ and $c_2 = 1/4$

Therefore, the unique solution of the given recurrence relation with the given initial conditions $a_0=1$, $a_1=2$ is: $a_n = \frac{3}{4}3^n + \frac{1}{4}(-1)^n = \frac{3^{n+1}+(-1)^n}{4}$

Case 2: $a_n = 6a_{n-1} - 9a_{n-2}$

$$P(x) = r^2 - 6r + 9 = (r-3)^2 \quad a_n = c_1 3^n + c_2 n3^n$$

The unique solution is obtained by finding c 1and c 2using the initial conditions:

For n=1 and $a_1=3$, we get: $c_1 3^1 + c_2(1)(3)^1 = 3$ or $3c_1 + 3c_2 = 3$

For n=2 and $a_2=27$, we get: $c_1 3^2 + c_2(2)(3)^2 = 27$ or $9c_1 - 18c_2 = 27$

Solving the system of equations with two unknowns c_1 and c_2 gives:

$c_1 = -1$ and $c_2 = 2$

Therefore, the only solution of the recurrence relation with the given initial conditions is:

$$a_n = -3^n + 2n3^n = 3^n(2n - 1)$$

More generally, if r is a root of the characteristic polynomial and has multiplicity m then the general solution will have the form: $a_n = c_1 r^n + c_2 n r^n + c_3 n^2 r^n + \ldots + c_{m-1} n^{m-1} r^n = 0$

Case 3: $a_n = 2(a_{n-1} - a_{n-2})$ [14]

The characteristic polynomial of the recurrent equation is $r^2 - 2r + 2 = 0$ the solutions to this polynomial are $r_{1,2} = 1 \pm i$.

The general solution will be: $a_n = c_1(1+i)^n + c_2(1-i)^n$

In order not to leave the previous expression expressed in terms of complex numbers, we proceed to apply the procedure to take from a complex number to an expression that offers real numbers. Some elements referring to the complex numbers and using DeMoivre's theorem, can be obtained:

$$a + bi = \rho(\cos \Phi + i\,\mathrm{sen}\Phi) \quad \rho = \sqrt{a^2 + b^2}$$

$\tan\Phi=b/a$ $(\cos\Phi + i\,sen\Phi)^n=(\cos n\Phi + i\,sen\,n\Phi)$

Then, applying the above we have:

$$1+i=\sqrt{2}(\cos(\pi/4) + i\,sen(\pi/4))$$

$$1-i=\sqrt{2}(\cos(\pi/4) - i\,sen(\pi/4))$$

$$a_n=c_1(\sqrt{2}(\cos(\pi/4) + i\,sen(\pi/4)))^n + c_2(\sqrt{2}(\cos(\pi/4) - i\,sen(\pi/4))^n$$

$$=c_1((\sqrt{2})^n(\cos(n\pi/4) + i\,sen(n\pi/4))) + c_2((\sqrt{2})^n(\cos(n\pi/4) - i\,sen(n\pi/4)))$$

Once we have reached this point we proceed as in the previous cases if the equation is subject to initial conditions.

Method Change of variable: $T(n)=4T(n/2)+n$ for $n>1$

Here we replace n by 2^k (i.e. $k=\lg n$) to obtain $T(2^k)=4T(2^{k-1})+2^k$.

This can be written $t_k=4t_{k-1}+2^k$, if $t_k=T(2^k)=T(n)$. We now know how to solve this new recurrence, the characteristic equation is $(x-4)(x-2)=0$ and therefore:

$$t_k=c_1 4^k+c_2 2^k$$

Passing back to n in place of k we find $T(n)=c_1n^2+c_2n$

Iterative Method: Here we use the iterative method to solve the recurrence equation.

$$t_n=t_{n-1} + 3,\ n>1 \quad (1)$$

subject to the initial condition $t_1=2$.

Replacing n with n-1 in (1) we get:

$$t_{n-1}=t_{n-2} + 3$$

if we substitute this expression in (1) we get:

$$t_n=t_{n-2} + 3 + 3$$

$$t_n=t_{n-2} + 2\cdot 3 \quad (2)$$

Replacing n with n-2 in (1) we get:

$$t_{n-2}=t_{n-3} + 3$$

if we substitute this expression for t_{n-2} in (2) we get:

$$t_n=t_{n-3} + 3 + 2\cdot 3=t_{n-3} + 3\cdot 3$$

In general, we have:

$$t_n=t_{n-k} + k\cdot 3$$

If we make k=n-1 in this last expression we have:

$$t_n=t_1 + (n-1)\cdot 3$$

as t 1=2, we obtain the explicit formula:

$$t_n=2 + 3\cdot(n-1)$$

It is important to note that the general solutions that are proposed for a recurrent equation coincide with the general solutions that must be established to give a solution to a differential equation, it must even be added that if there are no initial conditions then the number of values to be obtained by the sequence is infinite.

Teaching task 1: Deer population

Link the application of recursive relationships to the growth of an animal species that may be endangered.

In this problem we make use of the contents previously discussed. In this example the iterative method is a viable option to solve the problem, but we have to take something into account when we are going to use it: the iterative method is used when the growth of the function is fixed and does not depend on another factor, that is to say the function we are measuring grows in a size t (fixed) in the instant of time t_0, t_1,..., t_k.

Assume that the deer population in a reserve is 1000 at time n=0 and that the increase between time n-1 and time n is 10% of the size at time n-1. Write a recurrence relation and an initial condition that defines the deer population at time n and then solve for the recurrence relation (Johnsonbaugh, 2018).

Let d_n be the population of deer at time n. We have the initial condition $d_0=1000$. The increase from time n-1 to time n is d_n-d_{n-1}. Since this increase is 10% of the size at time n-1, we obtain the recurrence relation:

$$d_n-d_{n-1}=0.1d_{n-1}$$

Solving the above equation by iterations we get:

$$d_n=1.1d_{n-1}=1.1(1.1d_{n-2})=(1.1)^2(d_{n-2})=\cdots=(1.1)^n d_0=(1.1)^n 1000.$$

Teaching task 2: Another deer population problem

Assume that the Rustic County deer population is 200 at time n=0 and 220 at time n=1, and that the increase from time n-1 to time n is twice the increase from time n-2 to time n-1. Write the recurrence relation and an initial condition that defines the deer population at time n and then solve for the recurrence relation [15].

$$d_1=200\ d_2=220$$

The increment from time n-1 to time n is $d_n - d_{n-1}$, and the increment from time n-2 to time n-1 is $d_{n-1} - d_{n-2}$. Then we obtain the recurrence relation $d_n - d_{n-1}=2(d_{n-1} - d_{n-2})$, which is written as $d_n=3d_{n-1} - 2d_{n-2}$. The solution of the above recurrence is:

$$d_n=180 + 20 - 2n$$

Teaching task 3: Tree planting

Link the application of recursive relationships to flora.

In a plantation there are 51 rows of trees. Each row has two more trees than the row before it. Row 26 has 57 trees. How many trees are in the first and last rows?

$$f_n=f_{n-1}+2\ f_{26}=57$$

Substitute n for n-1 and you get:

$$f_{n-1}=f_{n-2} + 2\ (**)$$

Substitute ** into the original and you get:

$$f_n=f_{n-2} + 2 +2$$

Substitute n for n-2

$$f_{n-2}=f_{n-3}+2 \text{ (***)}$$

Substitute the above equation and you get:

$$f_n=f_{n-3}+2+2+2=f_{n-3}+6$$

Using the iterative method, the following is obtained in a general way:

$$f_n=f_{n-k}+2k \text{ it is known that } n-k=26 \rightarrow -k=26-n$$

Substituting the value of -k into the general equation, we get:

$$f_n=57+2(26-n)=109-2n \rightarrow f_0=109,\ f_1=107,\ f_2=105 \text{ y } f_{49}=11\ f_{50}=9$$

$$\rightarrow f_1=107,\ f_2=105 \text{ y } f_{50}=9\ f_{51}=7$$

The obtained solutions are correct, some authors define that the first term of a sequence is the element s0 while others say that it is s1, that's why in the previous example the two possible ways in which the terms of a sequence can be expressed were proposed.

Other teaching tasks

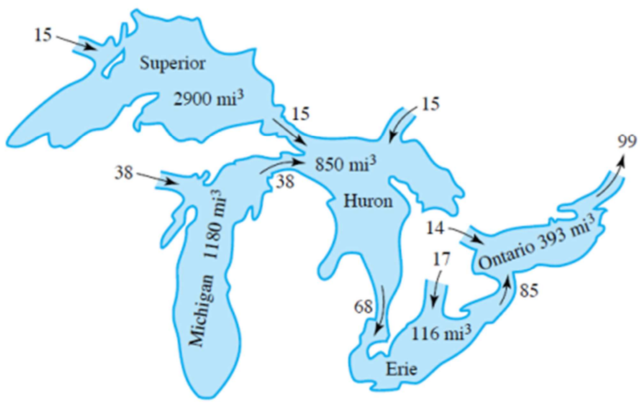
These can be done during a class or as independent study activities.

1. The equations model aphid (A) and ladybird (L) populations.

$$\frac{dA}{dt} = 2A - 0.01AL$$

$$\frac{dL}{dt} = -0.5L + 0.0001AL$$

   a) Find the equilibrium solutions and explain their meaning.
   b) Find an expression for dL/dA.
2. In March 1976, the world population reached 4 billion. One magazine predicted that with an average annual growth rate of 1.8%, the world population would be 8 billion at the end of 45 years. How does this value compare with that predicted by the model that the growth rate is proportional to the population at any point in time?
3. Set up a mathematical model for how long it takes to clean up the Great Lakes. Use the image below to help you.
4. Studies have shown that the Mediterranean fruit fly grows in proportion to the number present at any given time. After 2 hours of observation 800 families of the fly are formed and after 5 hours 2 000 families are formed. Find: *a*. The equation representing the number of families as a function of time, and *b*. the number of family that were present at the start.
5. The population of a certain city increases proportionally to the number of inhabitants that there are at a given time in it. If after 5 years, the population has tripled and after 8 years it is 45 000 inhabitants, find the number of citizens that were there initially.
6. In the breathing process we absorb air containing mainly nitrogen and oxygen, and when we exhale we give off carbon dioxide. We want to purify the atmosphere in a hall where a large number of people are dancing; to do this, a stream of pure air of 3 500 m^3/h of air is passed through, which we will call *Qa1*, and 3 000 m^3/h of polluted air (*Qa2*), containing carbon dioxide, is blown out. It is known that the volume of the room is 10 000 m^3 and that the initial concentration of carbon dioxide in the room is 0.1% of the room volume. Assuming that the density remains constant, what is the concentration of carbon dioxide, $C\ f\ CO_2$, 4 hours after the dance has started? The concentration is expressed in g/m^3.
7. The growth rate of a population is proportional to the number of its inhabitants. If after 18 years the population has doubled and after 25 years the population is 200000, find:
   a. The initial number of inhabitants.
   b. How many inhabitants will it have in 100 years?
8. In a certain zoo it has been observed that the number of animals is increasing in proportion to the present number of animals. If after 5 years their number has doubled and after 7 years the number of animals is 576, find the number of animals on the day of the inauguration of the zoo.
9. A certain man has a couple of rabbits in a closed place and he wants to know how many will be able to reproduce in a year from the initial couple, taking into account that in a natural way they have a couple in a month, and that from the second month they start to reproduce. Establish a recurrent relationship for the previous problem and solve it. Try to define a differential equation that solves the same problem and compare the results.



Note: Only 4 hypotheses are needed.

Figure 1. *The Great Lakes.*

4. Conclusions

The teaching tasks allow the approach and solution of mathematical-environmental problems, in particular practical problems, which require the application of knowledge from both disciplines.

A teaching task alone does not allow the student to understand the essence of the process of solving the problems of everyday life, much less to get to form a more global vision of the object of study, for this requires real teaching tasks, which the teacher must know how to design.

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